Solution:
Let $T : P_n \to P_n$ be the transformation defined by $T(p(x)) = x^n p\left(\frac{1}{x}\right)$.

i) Show that T is linear.
Closure under Addition. Let $f(x), g(x) \in P_n$ and consider

$$\begin{aligned}
T(f(x)+g(x)) &= T((f+g)(x)) \\
&= x^n (f+g)\left(\frac{1}{x}\right) \\
&= x^n \left(f\left(\frac{1}{x}\right) + g\left(\frac{1}{x}\right)\right) \\
&= x^n f\left(\frac{1}{x}\right) + x^n g\left(\frac{1}{x}\right) \\
&= T(f(x)) + T(g(x))
\end{aligned}$$

which shows that T is closed under addition.
Closure under Scalar Multiplication. Let $f(x) \in P_n$ and $c \in \mathbb{R}$. Consider

$$\begin{aligned}
T(c \cdot f(x)) &= T((cf)(x)) \\
&= x^n (cf)\left(\frac{1}{x}\right) \\
&= x^n \left(c \cdot f\left(\frac{1}{x}\right)\right) \\
&= c\left(x^n f\left(\frac{1}{x}\right)\right) \\
&= c \cdot T(f(x))
\end{aligned}$$

which shows that T is closed under scalar multiplication.
Therefore, T is a linear transformation, as required.

ii) Show that T is an isomorphism.
Since T is linear and the dimensions of both the domain and codomain are $n+1$, it follows that T is an isomorphism if and only if T is one-to-one or onto.
We will show that T is onto. Let $p(x) \in P_n$. Then $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ for some constants $a_0, a_1, \ldots, a_n \in \mathbb{R}$. Pick $q(x) = a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1}x + a_n$. Notice that $q(x) \in P_n$.
Now observe that

$$\begin{aligned}
T(q(x)) &= x^n \cdot q\left(\frac{1}{x}\right) \\
&= x^n \cdot \left(a_0 \cdot \left(\frac{1}{x}\right)^n + a_1 \cdot \left(\frac{1}{x}\right)^{n-1} + \cdots + a_{n-1} \cdot \left(\frac{1}{x}\right) + a_n\right) \\
&= x^n \cdot \left(a_0 \cdot \left(\frac{1}{x^n}\right) + a_1 \cdot \left(\frac{1}{x^{n-1}}\right) + \cdots + a_{n-1} \cdot \left(\frac{1}{x}\right) + a_n\right) \\
&= a_0 \cdot \left(\frac{x^n}{x^n}\right) + a_1 \cdot \left(\frac{x^n}{x^{n-1}}\right) + \cdots + a_{n-1} \cdot \left(\frac{x^n}{x}\right) + a_n \cdot x^n \\
&= a_0 + a_1 x + \cdots + a_{n-1}x^{n-1} + a_n \cdot x^n \\
&= p(x)
\end{aligned}$$

which shows that $p(x)$ has a preimage. Since $p(x)$ was arbitrary, it follows that T is onto.
Therefore, T is an isomorphism as required.