Remark: Before we proceed with the solution, note that since we are combining two standard decks, we have 104 cards total. Also, note that the number of ranks and suits do not change. There are still 13 ranks and 4 suits. What does change, however, is the number of cards for each rank. There will be 8 cards for each of the 13 ranks as opposed to 4 cards for every rank.

Solution for Part A: A *full house* is a five card hand which contains 3 cards of one rank and 2 cards of another rank. Thus, the operation of creating full house hands can be broken down into the following steps:

Step 1: Pick a rank. There are $\binom{13}{1}$ ways to do this.

Step 2: Pick three cards from that chosen rank.
Let S, H, C, D denote Spades, Hearts, Clubs, and Diamonds respectively. So we need to pick three cards from the following:

$$S, S, H, H, C, C, D, D$$

We need to consider two cases:

- (No duplicate pairs). In this case we want to pick three elements from the set $\{S, H, C, D\}$. This can be done in $\binom{4}{3}$ ways.
- (One duplicate pair). In this case suppose we picked S, S. Now we just need one more element from the set $\{H, C, D\}$. This can be done in $\binom{3}{1}$ ways. Since the duplicate cards could have been H, H, C, C, or D, D we have a total of $4\binom{3}{1}$ ways.

Since these cases form a partition, the sum rule applies, and so the total number of ways to pick three cards from the first rank is

$$\binom{4}{3} + 4\binom{3}{1} = 16$$

Step 3: Pick a different rank. There are $\binom{12}{1}$ ways to do this.

Step 4: Pick two cards from the newly chosen rank.
Again, let S, H, C, D denote Spades, Hearts, Clubs, and Diamonds respectively for the chosen rank. So we need to pick two cards from the following:

$$S, S, H, H, C, C, D, D$$

We need to consider two cases:

- (No duplicate pairs). In this case we want to pick two elements from the set $\{S, H, C, D\}$. This can be done in $\binom{4}{2}$ ways.
- (One duplicate pair). In this case we can pick S, S, H, H, C, C, or D, D. In other words, there are $\binom{4}{1}$ ways to pick an element from $\{S, H, C, D\}$ to duplicate.

Since these cases form a partition, the sum rule applies, and so the total number of ways to pick two cards from the second rank is

$$\binom{4}{2} + \binom{4}{1} = 10$$

Thus, by the product rule, there are a total number of

$$\binom{13}{1} \cdot 16 \cdot \binom{12}{1} \cdot 10 = 24960$$

full houses.

Solution for Part B: A *fuller house* is a twelve card hand which contains 6 cards of one rank, 4 cards of another rank, and 2 cards of another different rank. Thus, the operation of creating fuller house hands can be broken down into the following steps:

Step 1: Pick a rank. There are $\binom{13}{1}$ ways to do this.

Step 2: Pick six cards from that chosen rank.
Let S, H, C, D denote Spades, Hearts, Clubs, and Diamonds respectively for the chosen rank. So we need to pick six cards from the following:

$$S, S, H, H, C, C, D, D$$

We need to consider two cases:

- (Two duplicate pairs). In this case we want to pick two elements from the set $\{S, H, C, D\}$ that we want to duplicate. This can be done in $\binom{4}{2}$ ways. We still need to select two more cards. There are two suits left. So this can be done in $\binom{2}{2}$ ways.
- (Three duplicate pairs). In this case we want to pick three elements from the set $\{S, H, C, D\}$ that we want to duplicate. This can be done in $\binom{4}{3}$ ways.

Since these cases form a partition, the sum rule applies, and so the total number of ways to pick three cards from the first rank is

$$\binom{4}{2}\binom{2}{2} + \binom{4}{3} = 10$$

Step 3: Pick a different rank. There are $\binom{12}{1}$ ways to do this.

Step 4: Pick four cards from the newly chosen rank.
Again, let S, H, C, D denote Spades, Hearts, Clubs, and Diamonds respectively for the chosen rank. So we need to pick four cards from the following:

$$S, S, H, H, C, C, D, D$$

We need to consider three cases:

- (No duplicate pairs). In this case we want to pick four elements from the set $\{S, H, C, D\}$. This can be done in $\binom{4}{4}$ ways.
- (One duplicate pair). In this case suppose we picked S, S. Now we just need two more elements from the set $\{H, C, D\}$. This can be done in $\binom{3}{2}$ ways. Since the duplicate cards could have been H, H, C, C, or D, D we have a total of $4\binom{3}{2}$ ways.
- (Two duplicate pairs). In this case pick the two suits from $\{S, H, C, D\}$ that will be duplicated. This can be done in $\binom{4}{2}$ ways.

Since these cases form a partition, the sum rule applies, and so the total number of ways to pick three cards from the first rank is

$$\binom{4}{4} + 4\binom{3}{2} + \binom{4}{2} = 22$$

Step 5: Pick a different rank. There are $\binom{11}{1}$ ways to do this.

Step 6: Pick two cards from the newly chosen rank.

Again, let S, H, C, D denote Spades, Hearts, Clubs, and Diamonds respectively for the chosen rank. So we need to pick two cards from the following:

$$S, S, H, H, C, C, D, D$$

We need to consider two cases:

- (No duplicate pairs). In this case we want to pick two elements from the set $\{S, H, C, D\}$. This can be done in $\binom{4}{2}$ ways.
- (One duplicate pair). In this case we can pick S, S, H, H, C, C, or D, D. In other words, there are $\binom{4}{1}$ ways to pick an element from $\{S, H, C, D\}$ to duplicate.

Since these cases form a partition, the sum rule applies, and so the total number of ways to pick two cards from the second rank is

$$\binom{4}{2} + \binom{4}{1} = 10$$

Thus, by the product rule, there are a total number of

$$\binom{13}{1} \cdot 10 \cdot \binom{12}{1} \cdot 22 \cdot \binom{11}{1} \cdot 10 = 3775200$$

fuller houses.